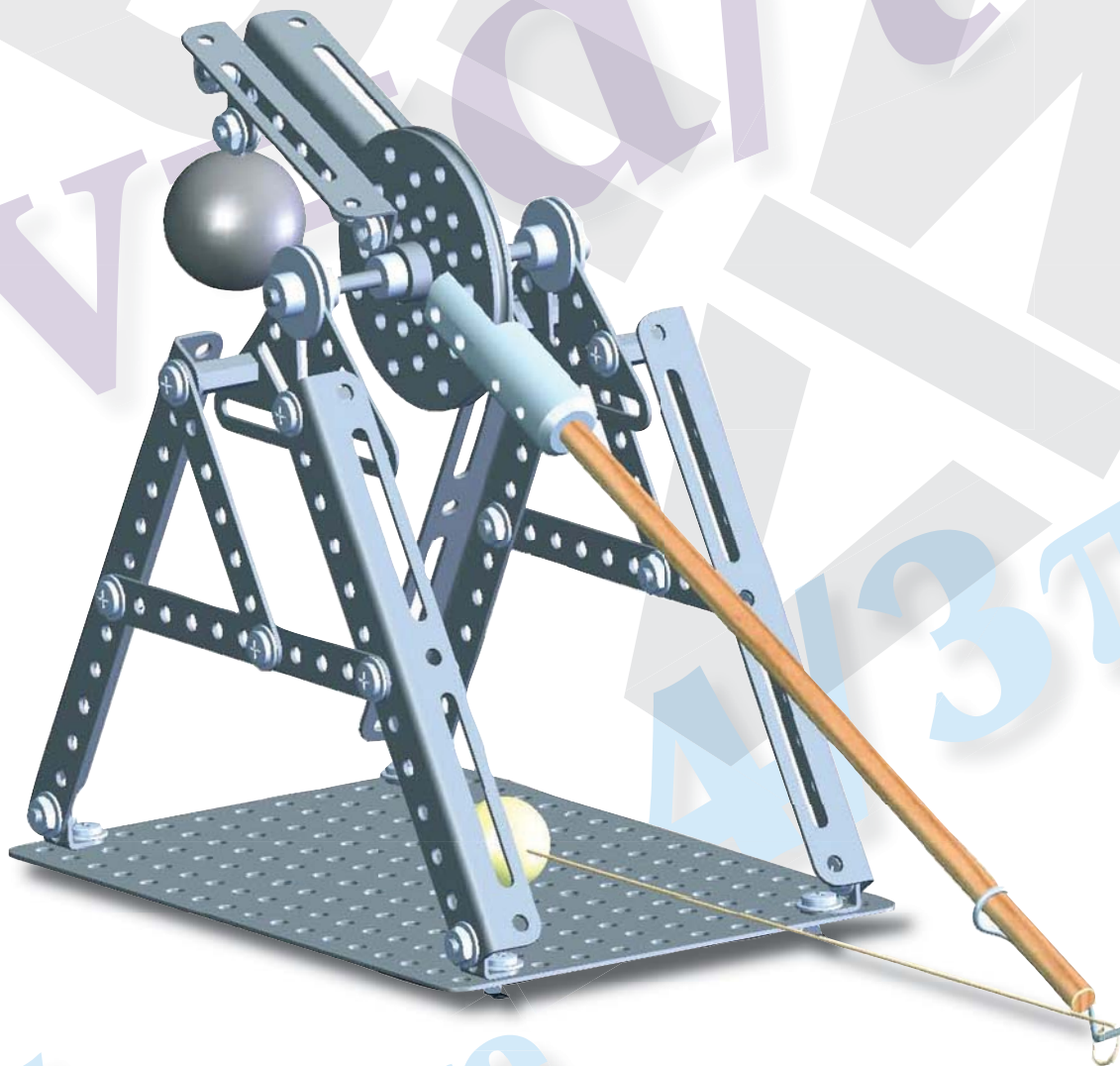

Totally Trebuchet



Gears

Educational Systems, LLC

www.gearseds.com

Totally Trebuchet **Overview**

Introduction

The GEARS-EDS Totally Trebuchet Program combines SolidWorks® with STEM based lessons and activities. The comprehensive Totally Trebuchet Program described on these two pages is available from GEARS-EDS at <http://www.gearseds.com/> or from SolidWorks resellers. Included on the disk is a small sampling of our math, science and engineering curriculum.

Trebuchets are fun to design, build and use. The Totally Trebuchet curriculum is a suite of self-paced tutorials, teacher-led lessons and project-based activities. This educational program provides an organized structure in which students and teachers acquire engineering skills and competencies through the design, construction, refinement and competitive use of model trebuchets. Totally Trebuchet lessons and activities demonstrate how math, science and engineering skills can be applied to the solution of an open-ended engineering challenge.

Content

Totally Trebuchet is an interdisciplinary engineering education exercise in which teams of students develop and employ transferable skills and competencies in one or all of the following areas:

- Science
- Engineering
- Mathematics
- Engineering Drawing
- Communication and Careers

The Build Report / Notebook

The build report is the essential element or “Deliverable” expected from students who participate in this program of engineering education lessons and activities. The Build Report includes:

- Solid Models, working drawings and COSMOS analysis that graphically describes trebuchet components and assemblies
- Research papers and presentations.
- Mathematical models used to describe or predict trebuchet parts or performance.
- Data from design iterations modeled on the TrebStar® trebuchet simulator.
- Spreadsheet analysis that includes the amounts, cost and weights of components and materials used to fabricate trebuchet parts and assemblies.
- Logs or diaries that chronicle the engineering design process which includes ideation, construction, testing and making the changes necessary to produce a working trebuchet.
- Class notes, teacher tests or quizzes, lesson worksheets and evidence of participation in the activities.

Assessment

Educational success is best evidenced by a student’s ability to clearly articulate what they know and are able to do. The measure of a student’s participation in the Totally Trebuchet program of activities is evaluated in these three ways: Demonstrations of their competencies and skills, the quality of their build report, and the caliber of their engineering performance in the Totally Trebuchet challenge games.

Totally Trebuchet

Educational Components

Totally Trebuchet is a complete set of educational tools and text materials designed to provide a comprehensive introduction to science, math and engineering studies (6 to 8 week program).

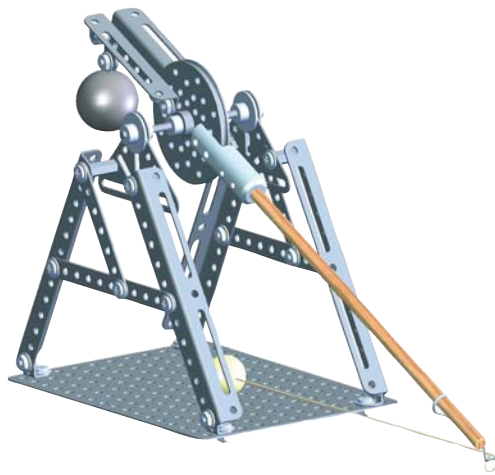
- **GEARS-IDS™ Trebuchet Kit and Construction Manual**
Rugged and re-configurable structural components designed to withstand the yearly demands of classroom service. These materials are organized in a storage container for easy classroom management.
- **TrebStar® Trebuchet Computer Simulation**
This easy to use simulation mathematically models trebuchet performance and efficiency. It allows teachers and students to quickly and accurately create dozens of trebuchet design iterations.
- **Will It Break? A Trebuchet Technical White Paper By Donald Siano PhD.**
A clear, concise introduction to the engineering and analysis needed to safely design and build small, medium and large-scale trebuchets. This is a great example of a technical white paper authored by a professional engineer with a passion for hurling!
- **GEARS-IDS™ Totally Trebuchet Curriculum**
An integrated program of standards based science, mathematics and engineering studies organized around an engineering challenge. Students work in teams to produce an engineering build report that chronicles the design, construction and competitive use of working trebuchets. The curriculum materials are divided into seven chapters and mapped to national education standards.
 1. **Building the Trebuchet Team – SCANS - Tools for Success in Team Based Projects**
 2. **Engineering Communication - Researching Trebuchets**
 3. **Trebuchet Math**
 4. **Trebuchet Science**
 5. **Trebuchet Engineering**
 6. **Totally Trebuchet Games and Engineering Challenges**
 7. **Trebuchet Build Report / Notebook - The Purpose of Engineering**

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Sample Math Curriculum

Pages M1-M4



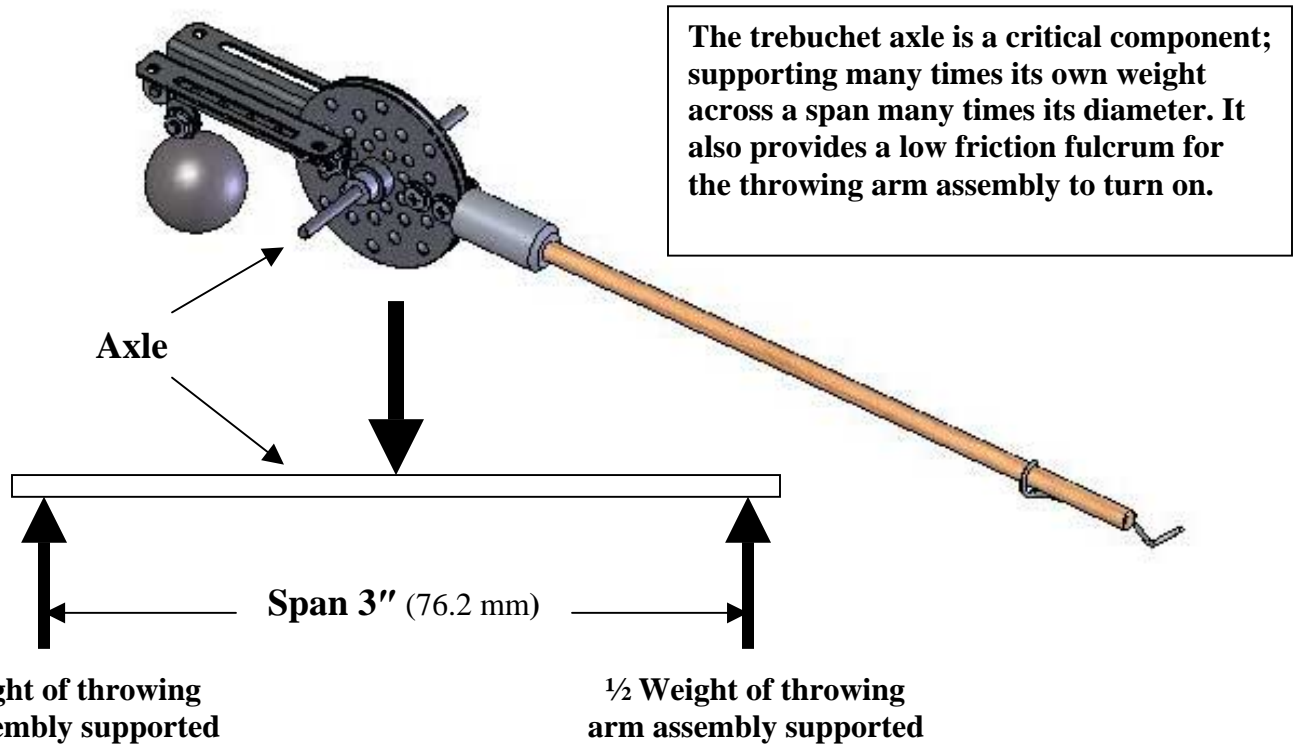
Perimeter, Area and Volume Activity

Describing and Visualizing Totally Trebuchet Geometry

Activity #1 The Axle – Cross Section and Span

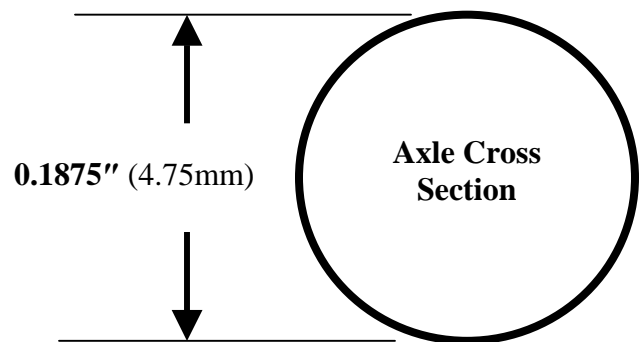
Sketch the GEARS trebuchet axle and label the dimensions on a separate sheet of paper. Remember to include this sketch and all accompanying calculations and answers in your build report.

Throwing arm assembly



Determine the following values:

1. The axle length (inches and centimeters)
2. Axle cross sectional area (in² and cm²)
3. Axle weight (pounds ounces and grams)
4. Span divided by the cross section diameter
5. Volume of the cylindrical axle. (in³ and cm³)



Perimeter, Area and Volume Activity

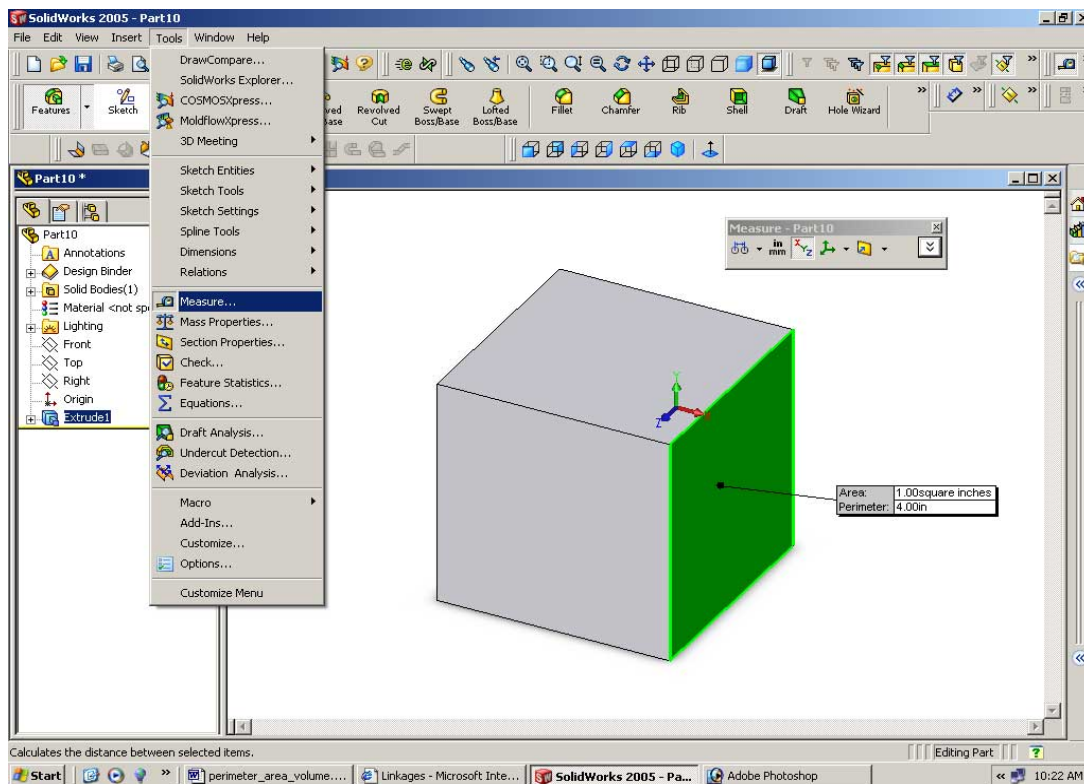
Describing and Visualizing Totally Trebuchet Geometry

Activity #2 Investigating SolidWorks - Geometry and Visualization

Powerful geometry tools are built into SolidWorks design software. They can be accessed using the TOOLS section in the standard toolbar menu. The tools used in this activity are MEASURE and MASS PROPERTIES.

Directions: Practice using the MEASURE and MASS PROPERTIES tools.

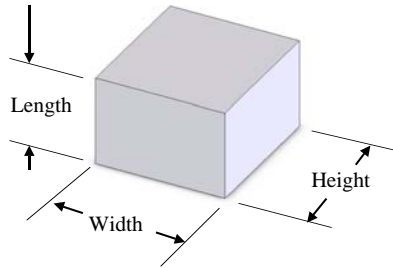
1. Open SolidWorks and create a 1" (25.4mm) cube.
2. Select a face (*Highlighted*) and open the TOOLS drop down menu.
3. Select Measure: The measure tool provides perimeter and area measures for that face.
4. Right click to deselect the face and measure tool.
5. Open the TOOLS drop down menu and select MASS PROPERTIES.
6. MASS PROPERTIES provides volume measurements for the part. This tool also includes center of mass (*relative to the origin*), and weight relative to the material density.
7. Use SolidWorks to construct a compound solid of your own design. Use the MEASURE tool to determine the area and perimeter of some or all of the faces.
8. Use the MASS PROPERTIES tool to determine the volume.
9. Print your work and write in the perimeter, area and volume information on the printed page.



Sample Math Curriculum

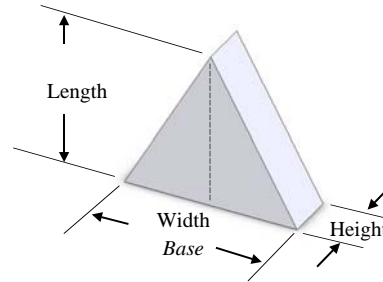
Volume Formulas for Solid Objects

$$\text{Volume} = (\text{Length} \times \text{Height}) \times \text{Width}$$



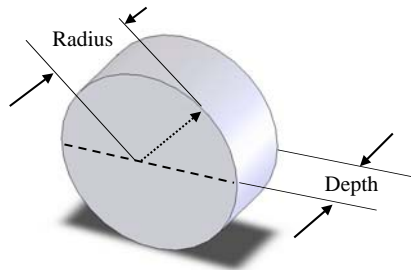
Volume of a Rectangular Prism

$$\text{Volume} = (\frac{1}{2}\text{Length} \times \text{Height}) \times \text{Width}$$



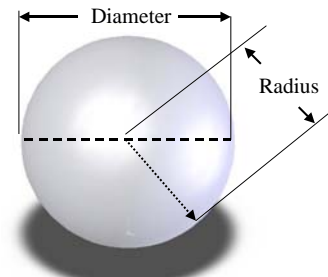
Volume of a Triangular Prism

$$\text{Volume} = (\pi \times \text{Radius}^2) \times \text{Depth}$$



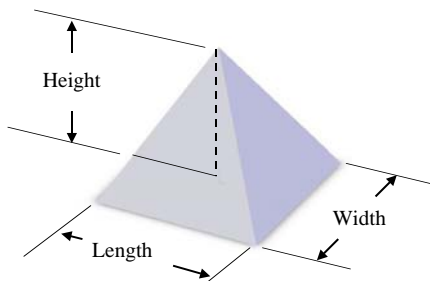
Volume of a Cylinder

$$\text{Volume} = \frac{4}{3} \pi \times \text{Radius}^3$$



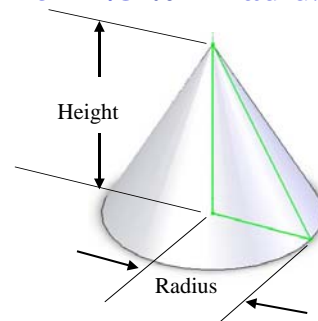
Volume of a Sphere

$$\text{Volume} = \frac{1}{3} (\text{Area Base} \times \text{Height})$$



Volume of a Pyramid

$$\text{Volume} = \frac{1}{3} \pi \times \text{Radius}^2 \times \text{Height}$$



Volume of a Cone

Assessment Rubric: Perimeter-Area-Volume

Describing and Visualizing Totally Trebuchet Geometry

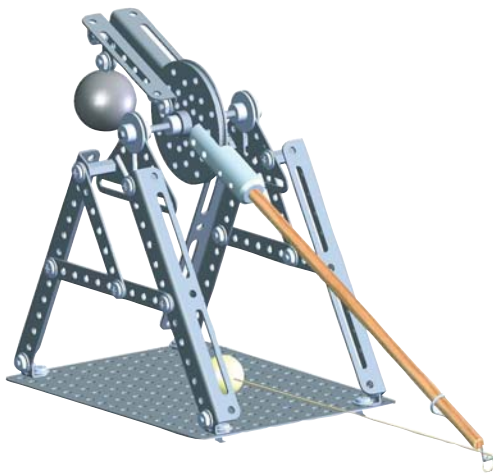
This rubric is offered as an example. Teachers and students are encouraged to create assessment tools that reflect their needs and expectations.

Required Skill or Competency	Meets or Exceeds this Requirement Score 4-5 pts	Meets Some of this Requirement Score 2-3 pts	Meets little or None of this Requirement Score 0-1 pts
Calculate the perimeter and areas of rectangles, parallelograms, triangles and circles.			
Calculate the volume of polyhedrons, cylinders, cones and spheres.			
Identify, describe and calculate the perimeter of selected Totally Trebuchet components.			
Identify, describe and calculate the area of selected Totally Trebuchet components.			
Identify, describe and calculate the volume of selected Totally Trebuchet components.			
Uses spreadsheets, SolidWorks or other technology to confirm the accuracy of the axle area or volume calculations performed in this lesson.			
Successfully completes the activities, worksheets and assessments assigned in this lesson.			
Maintains accurate and organized records of class notes, activities, worksheets and assessment tools used in this lesson.			



Sample Science Curriculum

Pages S1-S8



Lesson 2: Mass, Gravity, Weight and Density

Visualizing and Describing The Science of Totally Trebuchet

Mass is the measure of the amount of material in an object. It is the nature of all mass to be mutually attractive. This force of attraction is called **gravity**.

Gravity is proportional to mass. Therefore the most significant gravitational force is due to the earth itself.

Weight is the measure of the downward force created by the mutual gravitational attraction between the mass of an object and the mass of the earth.

Objects of the same size and shape (*volume*) can have different amounts of mass. The 1-1/2" (38mm) diameter lead sphere used as the GEARS trebuchet counterweight has significantly more mass than the (38mm) diameter wood sphere found in the kit. These spheres have equivalent volumes, but not equivalent masses. Since weight is the result of gravity acting on mass, the 1-1/2" (38mm) diameter lead sphere weighs more than the 1-1/2" (38mm) diameter wood sphere.

The lead sphere contains more mass per unit volume. Density is a measure of the mass per unit volume of an object. Weight can be substituted for mass when the density of an object or a substance is calculated. Units of mass and volume are kilograms per cubic meter (kg/m^3 in SI units) or in pounds per cubic foot (lbs/ft^3 in imperial units.)

In this lesson we will explore some fundamental properties of matter and materials in an effort to better understand and explain the operations of trebuchets and other mechanical systems.

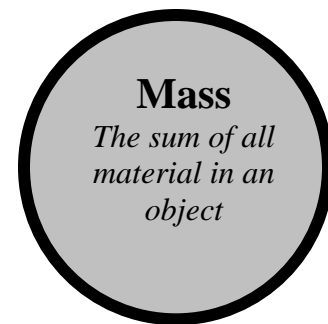
Mass

Mass is a fundamental **quantity** measured in kilograms or slugs.
SI units are kilograms (kg) **Imperial** units are slugs.

Mass is a scalar quantity, it can only be expressed as a magnitude. All objects have mass. Mass is a measure of the amount of matter or material an object contains.

Another example of a fundamental quantity is length. All objects have length.

An 8-inch long (203mm) pencil measures 8 inches long (203mm), anywhere in the universe. Length, mass and time are fundamental quantities that can be measured and compared irrespective of other factors or frames of reference (*Quantum mechanics and relativity excluded*).



Gravity

Gravity is a force of attraction that acts between all matter.

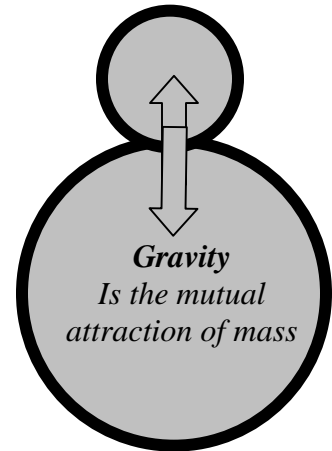
Gravity is measured by the rate at which it can accelerate mass.

SI acceleration is measured in meters per second per second (m/s^2). **Imperial** acceleration is measured in feet per second per second (ft/s^2).

- Gravity is a fundamental property of all mass. All mass has gravity all mass exerts gravitational forces that attract all other masses.
- The force of gravity is a vector quantity. It has both magnitude and direction.
- The force of gravitational attraction is proportional to the masses attracting one another. The more mass an object contains, the greater the gravitational force it exerts. Gravitation is additive.
- The earth is the most massive object that we interact with.
- The earth's mass and our mass exert mutually attractive forces that hold us in contact, or bring us back in contact when we attempt to leave the earth's surface.
- A bathroom scale provides a numerical measure of the force of gravity acting on our mass.
- If we increase our mass (Gain weight) the reading on the scale goes up. Conversely if we reduce our mass (Lose weight) the reading on the scale goes down.
- Since the scale is positioned between the earth and us, it records the change in the force of gravity acting between the earth and us.
- The gravitational attraction between two objects is proportional to their masses and inversely proportional to the square of their distance:

$$F = G \frac{m_1 m_2}{D^2}$$

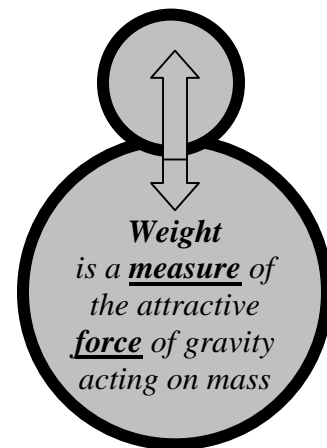
Where: F = Mutual gravitational force
G = Gravitational constant
D = Distance
m = mass



Weight

Weight is a derived measure of force. Weight is the result of the interaction **between mass and gravity**.

- Weight is not a fundamental quantity or force like mass or gravity.
- Weight is a force we can "feel" when we pick up an object.
- In the SI system, force is measured in units called Newtons.
- The force exerted by a mass under the influence of gravity is measured in Newtons.
- Weight is a measure of the effect of the force of gravity acting on the mass of an object.
- The measure of the weight of a given object can change with respect to other factors or frames of reference.



An 8-inch (203mm) pencil on earth, might feel like it weighs 8g (0.035oz.), but if you take that same pencil to the moon it might feel like it weighs only 1/6 of what it felt like on earth. In fact if you weighed it on both earth and the moon using a spring scale, you could prove this to be true. A spring scale measures force not mass.

Sample Science Curriculum

Gravity Revisited: A Force to Reckon With *(SI Units Only)*

Mass and weight are used interchangeably in daily life. For practical purposes the force of gravity over the earth's surface is considered to be constant. *(It actually varies with respect to altitude or distance from the earth's center and local variations in density.)* Weight and mass are equivalent expressions when the force of gravity is considered to be constant.

Galileo reasoned that gravity is a force that acts continuously on all objects. He experimentally determined that free falling masses do not fall at a constant velocity. Simply put, the force of gravity caused falling masses to accelerate, and this rate of acceleration could be measured.

Newton, who ironically was born on the day Galileo died, created the mathematical models to quantify and predict the implications of the gravitational forces that Galileo had measured.

Newton described the essential relationship between force, mass and acceleration and produced an elegantly simple statement that explains the movement of objects in the physical universe.

$$\text{Force} = \text{Mass} * \text{Acceleration}$$

Where:

Force is expressed in units called Newtons.

Mass is measured in Kilograms (*kg*).

Acceleration is expressed in meters/sec.² (m/s²).

Gravity x Mass = Weight

Newton's simple algebra provides a convenient way for us to differentiate between mass and weight. We can substitute the quantities above to predict the weight (Force) exerted by a given mass on earth.

$$\text{Weight} = \text{Mass} * \text{Acceleration of Gravity}$$

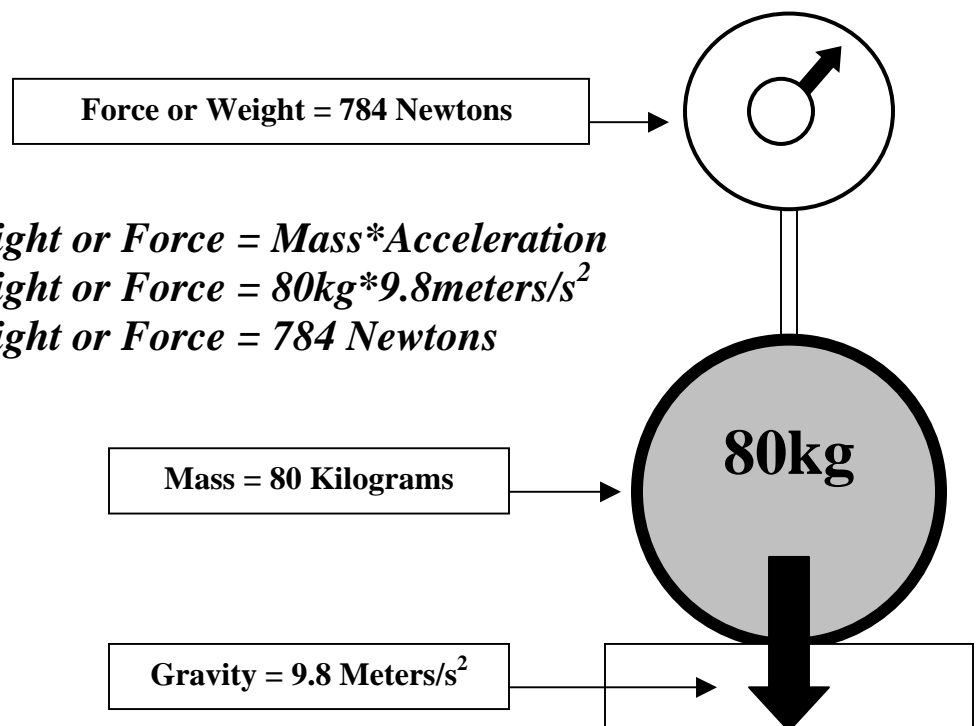
Where:

Weight is a downward force expressed in units called Newtons.

Mass is measured in Kilograms (*kg*).

Acceleration is expressed in meters/sec.² (m/s²).

$$\begin{aligned} \text{Weight or Force} &= \text{Mass} * \text{Acceleration} \\ \text{Weight or Force} &= 80\text{kg} * 9.8\text{meters/s}^2 \\ \text{Weight or Force} &= 784 \text{ Newtons} \end{aligned}$$



Sample Science Curriculum

Density

Density is a material property.

The density of an object varies as a ratio of mass to volume.

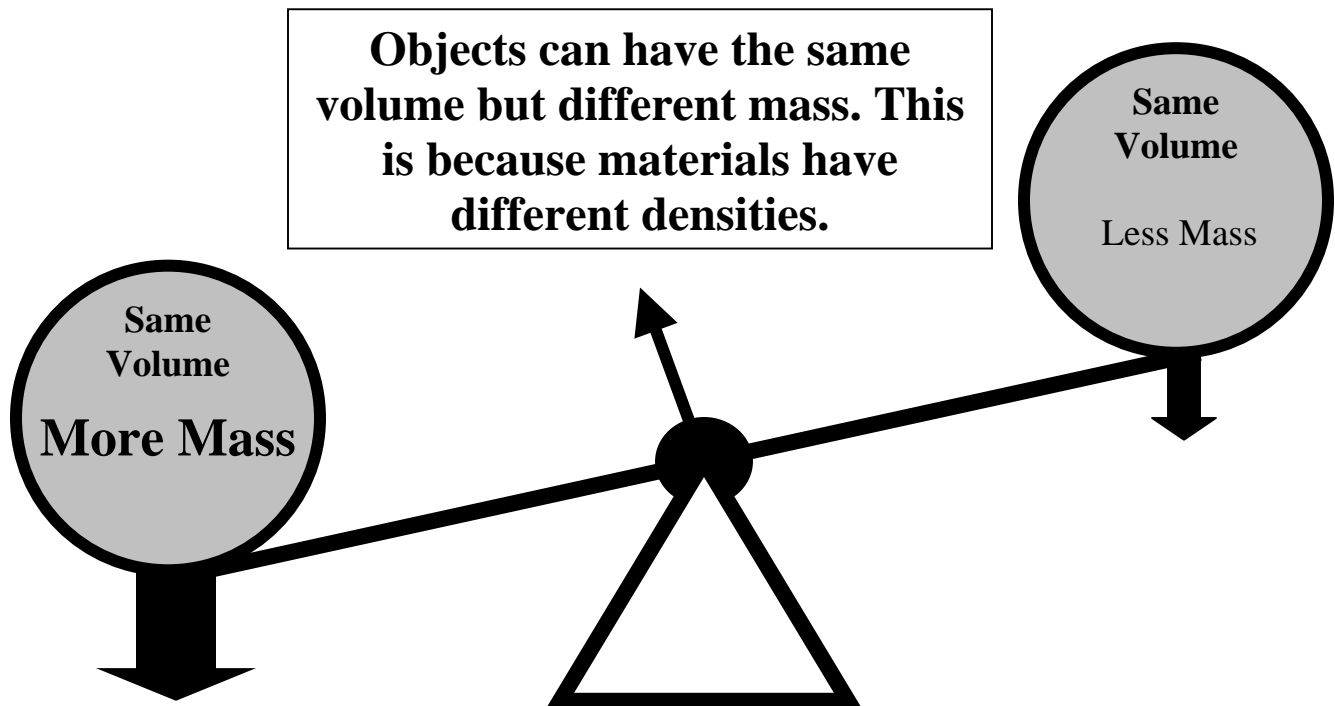
Specific materials have specific densities.

Density is a measure of the amount of mass contained in a unit of volume.

SI Units	Imperial Units	Example Material Densities
Kg/m ³	lbs/ft ³	Aluminum 2.7 g/cm ³
Kg/cm ³	lbs/inch ³	Steel 7.84 g/cm ³
g/cm ³	lbs/yd ³	Copper 8.85 g/cm ³
g/ml	oz/inch ³	Gold 19.32 g/cm ³
		Titanium 4.5 g/cm ³
		Polycarbonate 1.3 g/cm ³

The average density of an object is the combination of the densities and respective volumes of the different materials contained in an object. For example, automobile components are made of steel, glass, plastic, aluminum and dozens of other materials. If you computed the average density of a specific car, it would reflect the average densities and volumes of the materials from which that car was made. And you would not get an accurate measure of the “Density” of cars, because cars differ in the proportions of materials they use.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \quad \text{or} \quad \text{Density} = \frac{\text{Weight}}{\text{Volume}}$$



Activity #1: Calculate and Compare Density
Visualizing and Describing The Science of Totally Trebuchet

In this exercise we will use some of what we know about mass, volume and density to compute and compare the density of the 1-1/2" (38mm) diameter lead counterweight and wood sphere found in the GEARS Totally Trebuchet Kit.

Materials

Balance Scale 1-1/2" (38mm) lead counterweight and wood sphere. Calculator

Procedure

1. Record the mass of the spheres. This refers to using the gram scale to measure the mass of the spheres.

_____ **Mass** of 1-1/2" (38mm) lead sphere

_____ **Mass** of 1-1/2" (38mm) wood sphere

2. Determine the volume of the spheres. You can assume that both spheres are equal and their diameters are 1-1/2" (38mm). *

$$\text{Volume of a Sphere} = \frac{4}{3} \pi R^3$$

_____ **Volume** of 1-1/2" (38mm) lead sphere

_____ **Volume** of 1-1/2" (38mm) wood sphere

3. Calculate the densities of the spheres.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

_____ **Density** of 1-1/2" (38mm) lead sphere

_____ **Density** of 1-1/2" (38mm) wood (Maple) sphere

Example Material Densities		
Aluminum	2.7 g/cm ³	_____ Mass
Steel	7.84 g/cm ³	_____ Mass
Copper	8.85 g/cm ³	_____ Mass
Gold	19.32 g/cm ³	_____ Mass
Titanium	4.5 g/cm ³	_____ Mass
Polycarbonate	1.3 g/cm ³	_____ Mass
Wood (Maple)	_____	_____ Mass
Lead	_____	_____ Mass

4. Calculate the mass of a 1-1/2" (38mm) sphere made from the materials in the chart above. Complete the chart by adding the information in the blank areas.

* If you have dial calipers and experience using them, you can make accurate measures of the respective diameters of the wood and lead spheres in order to make a more accurate determination of their respective volumes.

Sample Science Curriculum

Work Sheet

Activity #2: Designing Counterweights

Visualizing and Describing The Science of Totally Trebuchet

Name _____ Date _____

Note: You will need to include and attach additional papers to complete this worksheet.

Sketches

Create a three-view sketch of the counterweight bucket design you choose to use. Include top, front and right side views. You will not know the dimensions until you have calculated the necessary amounts of counterweight material. Add dimensions later.

Density of Materials	SI Units (kg/m ³)	Imperial Units (lbs/ft ³)
Concrete with medium to heavy aggregate	_____	_____
Granite	_____	_____
Oak	_____	_____

Volume of Material Weighing Maximum 25,000 lbs or (11,339 kg)

This figure will be the Necessary Interior Bucket Volume.

	SI Units (m ³)	Imperial Units (ft ³)
Concrete with medium to heavy aggregate	_____	_____
Cut and stacked granite	_____	_____

Note: Cut and stacked granite will have voids amounting to 10% of the total granite volume. You will need to consider this when calculating the volume.

Determine the Necessary Dimensions (Length, Width and Height) for the Interior Cubic Shape of the Bucket.

	SI Units (meters)	Imperial Units (Feet)
Concrete with medium to heavy aggregate	L__W__H__	L__W__H__
Cut and stacked granite	L__W__H__	L__W__H__

Visualization

Provide three-view working drawings and solid model designs of the counterweight. Provide mathematical proof of the volume and weight of the bucket. Create your finished design and check your volume and weight using SolidWorks' mass properties tool.

Assessment Rubric: Mass, Density, Weight and Gravity
Describing and Visualizing the Science of Totally Trebuchet

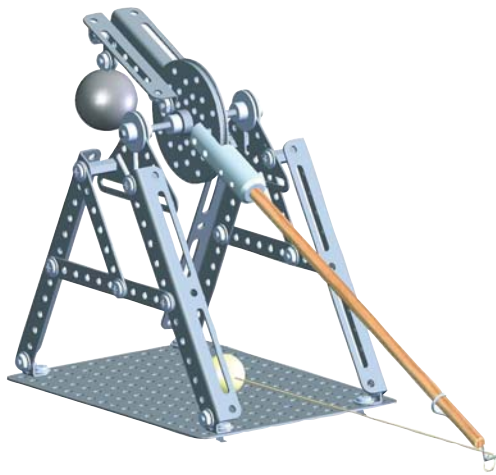
This rubric is offered as an example. Teachers and students are encouraged to create assessment tools that reflect their needs and expectations.

Required Skill or Competency	Meets or Exceeds this Requirement Score 4-5 pts	Meets Some of this Requirement Score 2-3 pts	Meets little or None of this Requirement Score 0-1 pts
Explain the difference between mass and weight.			
Calculate the weight of an object given the mass of the object.			
Research and record the density of different materials.			
Calculate the density of a material given the weight and volume.			
Calculate the volume of a material given the required weight and density.			
Calculate the volume and weight of an object given the dimensions and material density. Use SolidWorks' mass properties tool to find or check your answers.			
Calculate the volume and dimensions of a cube and sphere given the mass and required weight of a material.			
Maintains accurate and organized records of class notes, activities, worksheets and assessment tools used in this lesson.			
Successfully completes the Trebuchet counterweight design activity.			



Sample Engineering Curriculum

Pages E1-E10



Activity #9: Analyzing the Stress on the Trebuchet Axle Visualizing and Describing the Engineering of Totally Trebuchet

Engineers are responsible for designing and building machines, structures, processes, and systems that are dependable, reliable and unlikely to fail when used within the design limitations for which they were engineered. You might consider this the next time you are a passenger on a train, plane or automobile, or as you cross a bridge or ride an elevator.

Engineers use a variety of tools and techniques to help them create safe machines, structures, processes, and systems. Some of these tools include:

1. A working understanding of math and science principles.
2. Mathematical models.
3. CAD models (Static and dynamic).
4. Personal experience.
5. Physical models (Like the GEARS Totally Trebuchet model).
6. Prototypes.
7. Empirical testing and analysis.

The engineers and craftsmen who designed and built medieval trebuchets depended on personal experience, empirical testing and perhaps models and prototypes. We can make this assumption because there is no evidence they possessed the other engineering tools and skills listed above.

Objective

- Visualize and describe how cross sectional area and material strength affect the strength of the trebuchet axle.
- Calculate the maximum allowable stress (*material yield strength*) for a trebuchet axle with a given radius and material strength.

Terms and Concepts

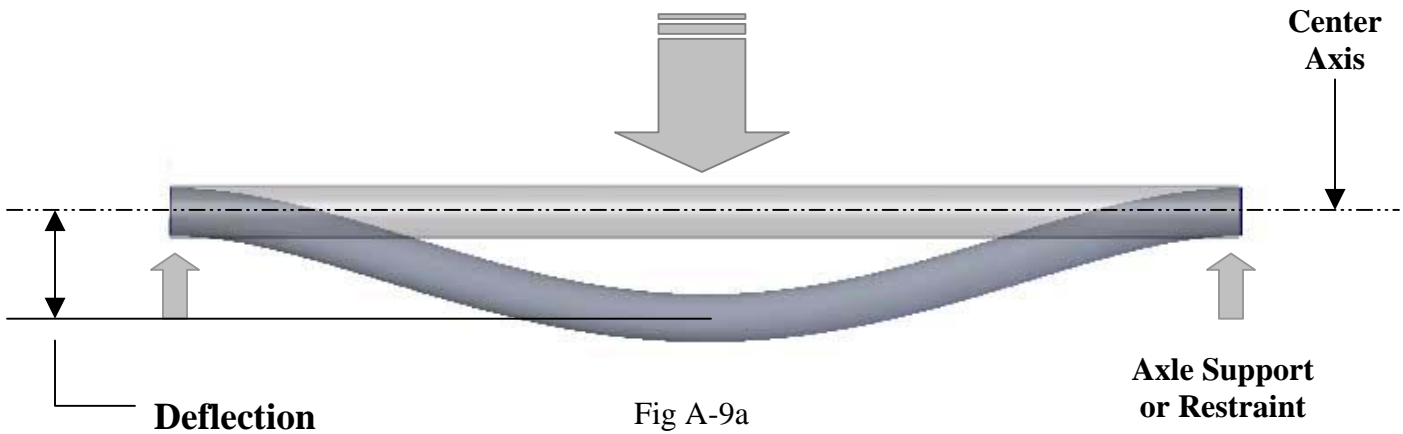
Use your research skills to help you develop a working understanding of these terms and concepts.

Strength of Materials Terms	Software and Hardware Tools	Engineering Terms
Compression	Calculators	Cross Sectional Area
Failure	COSMOSXpress	Deflect
Force per Unit Area	Dial Calipers	Deformation
Maximum Allowable Stress	SolidWorks	Dynamic Forces
PSI	Spreadsheet Software	Empirical Testing
Strain	GEARS-TrebStar	Flexural Formula
Stress		Load
Tension		Radius
Ultimate Strength		Span
Yield		Static Forces
Yield Strength		

Sample Engineering Curriculum

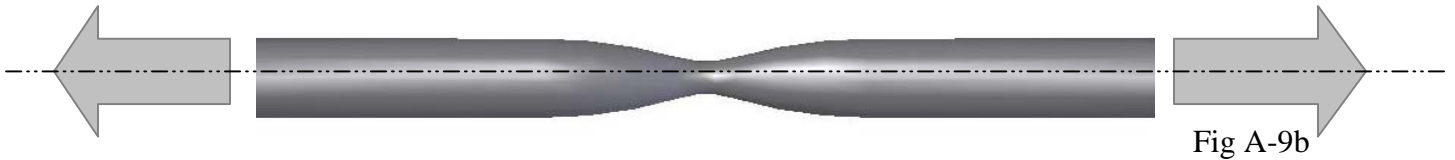
An Illustrated Look at Forces Acting on the Trebuchet Axle

A force that acts perpendicular to the center axis of the trebuchet axle will cause the axle to bend. This is called deflection. The amount of deflection is proportional to the applied force and inversely proportional to the cross sectional area of the axle. This simply means that if the axle is larger in diameter, it will support greater loads and deflect less.



Tension Force

Acts to stretch or pull the axle material apart. A rod failing under tension will "Neck Down" when pulled apart.



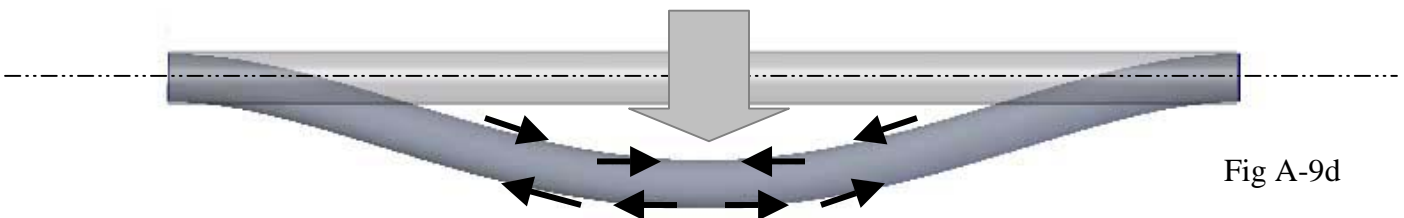
Compression Force

Acts to push the axle material together. A rod failing under tension will bulge if the compressive forces act along the center axis.



Tension and Compression

Occur simultaneously when the trebuchet axle is deformed. The material above the centerline is under compression while the material below the centerline is under tension.



Sample Engineering Curriculum

Force, Deflection, Stress and Yield

The loads or forces applied to the axle are called stress. Stress is a measure of force per unit area. An example might be pounds per square inch or Newtons per square centimeter. A given amount of stress (Force per unit area) will cause a given amount of change in the shape or length of the axle. This change in length is called strain. Put in plain terms, if you apply consistently greater force (Stress) the axle material will deform (Strain) to a greater degree.

Stretching a Steel Axle

Figure B9 is a graph illustrating the elongation (strain) of the axle due to increasing stress. This is a common way to test the strength of materials and components. In this test the axle is held securely on both ends. The axle is subjected to increasing tension (stretching) loads. The force of tension is divided by the cross sectional area of the axle. This yields the stress value in pounds per square inch (psi). Example:

A 0.250 inch diameter axle has an area = 0.05in^2 . If that axle is placed under a tension force of 1500 lbs, the stress would be:

$$\text{Stress} = \frac{1500 \text{ lbs.}}{0.05 \text{ in}^2} = 30,000 \text{ psi}$$

During the test the axle is stressed and the change in length or strain is measured. A plot of stress vs. strain is produced. The example graph indicates that stress is proportional to strain until the stress reaches approximately 30,000 psi. Beyond that point, an increase in stress produces an increasingly larger increase in strain. The material has begun to yield, and has suffered permanent deformation (stretch). The axle will no longer return to its original length when the applied stress is removed. The material has failed.

Engineers and designers do not like to design components to function at or even close to the yield point. It is common practice to design components with large enough cross sections to withstand two and even three times the applied loads without reaching the yield point.

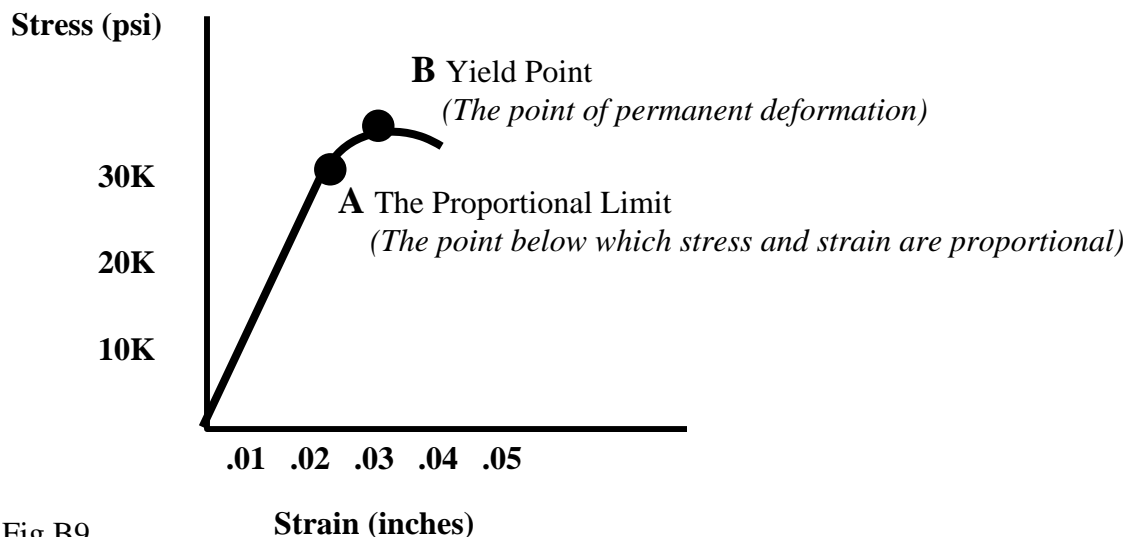


Fig B9

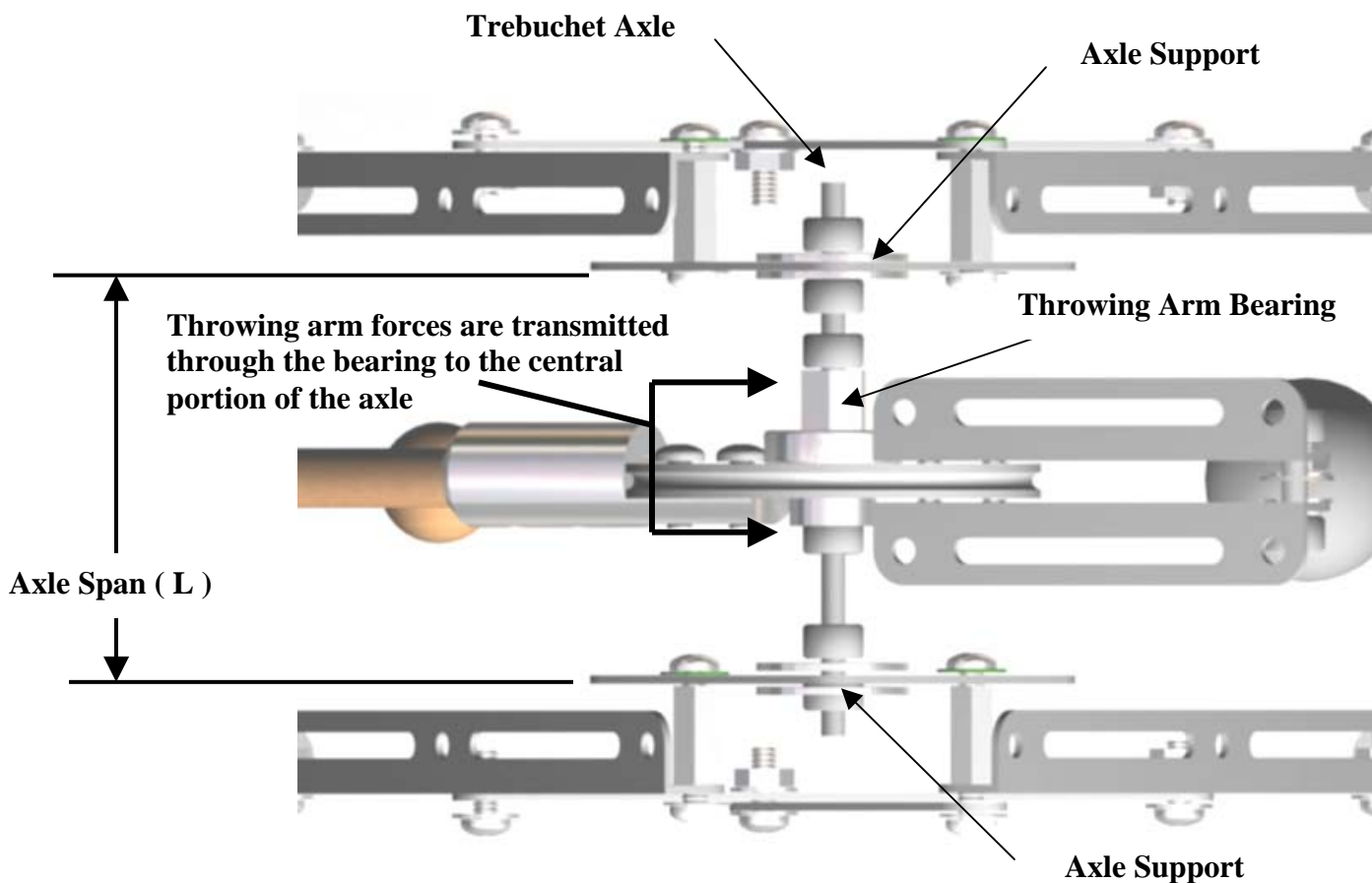
Sample Engineering Curriculum

The Trebuchet Axle

The trebuchet axle supports forces generated by the rotating counterweight, beam, and projectile. The axle is subject to significant dynamic (*due to motion*) forces during the throw and static (*stationary*) loads when it is at rest. These forces and loads act to bend (*deflect*) the axle. If the axle is not strong enough to support these loads, it will deflect causing permanent deformation of the axle. Permanent deformation means the molecular bonds holding the material together begin to fail and the material has reached or exceeded its yield strength. At this point the axle has failed, and is no longer safe to use. In a worse case, the axle will break, though permanent deformation is the more likely cause of failure.

Factor of Safety

In order to design a safe trebuchet axle it is necessary to consider the forces and loads to which the axle will be subject. A prudent engineer will design the axle to support loads equal to or greater than required. This additional design strength is called the “Factor of safety”. The factor of safety, or FOS, helps insure that components subject to extraordinary loads, or extended use, will not approach the yield point or fail under conditions for which they are designed. A common FOS is twice the design load. A component created with a FOS of 2, is designed to support a load equal to twice the maximum allowable load.



Top View of the GEARS Model Trebuchet

Will a Trebuchet Axle Fail?

The bending or breaking of the axle is likely to be the most frequent source of failure in a trebuchet system. In order to prevent axle failure, it is necessary to determine if the axle can support the loads imposed by the standing beam system as well as the more significant loads imposed by throwing.

It is possible to determine if the axle can support the loads in several different ways:

1. Trial and error. Build a trebuchet and use it. If the axle breaks, build another one. This is the costly, time consuming and inefficient method of axle design employed by ancient trebuchet builders.
2. Use the simplified flexural formula and the force information supplied by the GEARS-TrebStar simulator to develop a general approximation of the answer to the question: "Will it break?"
3. Use SolidWorks COSMOSXpress or other CAD modeling software to perform virtual tests of the axle's performance. This is a great way to obtain a "visual" understanding of how the axle will react to the applied forces.
4. Use all three methods. This is a great way to expand your engineering knowledge and skill, and arrive at a working solution to the question: "Will it break?"

Mathematical Model of the Axle

A trebuchet axle and the forces acting on it are illustrated in figure C9. This is a simplified example where the load on the axle is assumed to be acting on one point, at the center of the axle span. This is a "Worse case" scenario. An axle that can support the design load acting at the center of the span will likely support the loads generated by a working trebuchet. The actual loads of a working trebuchet will be distributed over a portion of the axle length and not applied directly to the center of the axle span.

The axle is assumed to be a rod with a span L , a diameter D , and supported on the ends, with a load P in the center of the beam. The load comes primarily from the weight of the counterweight plus the centrifugal force generated by the motions of the counterweight, beam and projectile during the throw.

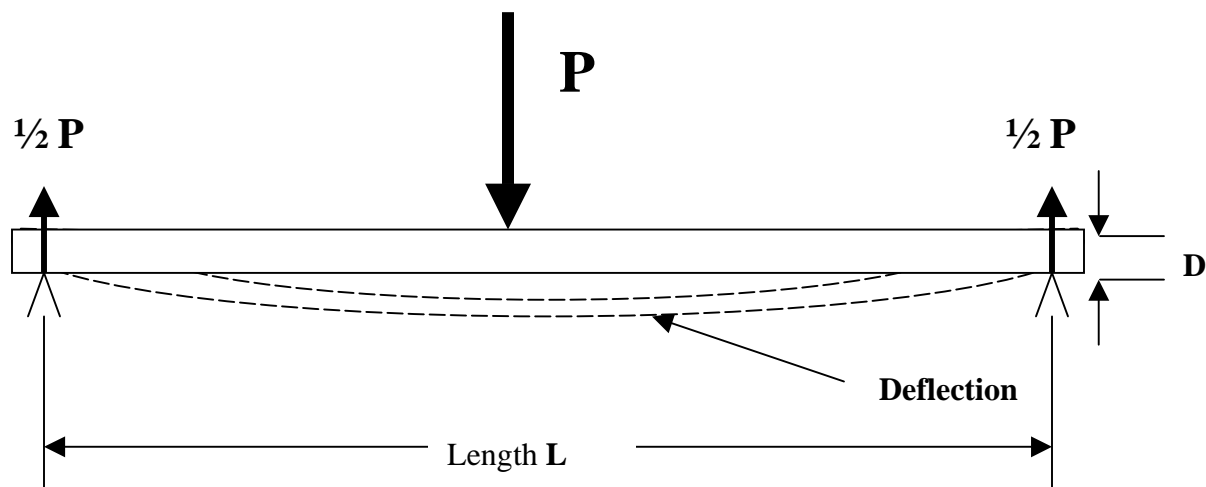


Fig. C9 An axle supported on two points equidistant from load P

Sample Engineering Curriculum

Analyzing the Stress on the Trebuchet Axle continued

For an axle with a given radius, and material: The greater the force P, and the span L, the more likely it is that the axle will fail. Conversely, if the span L is decreased, the axle will support greater loads.

In order to ensure that the axle will not deform (*bend*) beyond its yield point, it must be designed with an adequate cross section and material strength. The maximum load or stress that a beam or axle can support without suffering permanent deformation is called the maximum allowed stress. A reasonable value for the maximum allowed stress can be calculated using the simplified flexural formula show below.

The “Flexural formula” can be employed to calculate the:

1. Required material strength. (S_{\max})
2. Required axle radius. (r)
3. Required axle span. (L)

Flexural Formula

For an axle with a given circular cross-section, the yield strength or maximum allowed stress S_{\max} is:

$$S_{\max} = \left(\frac{1}{\pi}\right) * P * \frac{L}{r^3}$$

Where:

P = Load (*Force*) on the center of the axle

L = Span (*Distance between the supports*)

r = Radius of the axle

Using the Flexural Formula

Assume you are designing a model trebuchet with the following parameters:

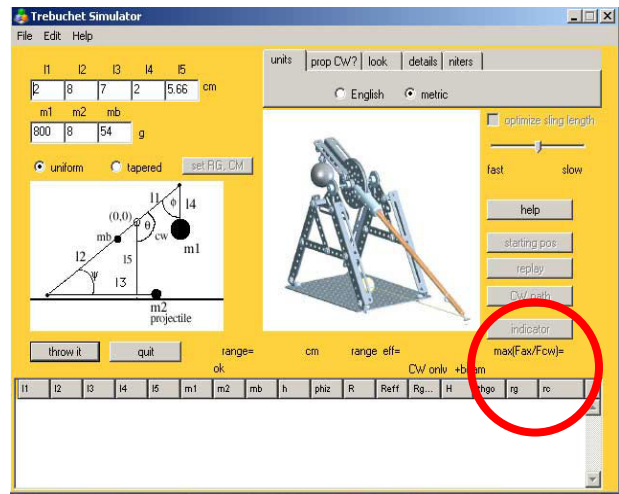
	SI Units	Imperial Units
m₁ Counterweight Mass	1000g	2.2lbs
m₂ Projectile Mass	20g	0.022lbs
mb Beam Mass	100g	0.22lbs
L₁ Counterweight Arm Length	10cm	0.33ft
L₂ Throwing Arm Length	45.5cm	1.50ft
L₃ Sling Length	37.5cm	1.23ft
L₄ Counterweight Pivot Arm	10cm	0.328ft
L₅ Axle Height	25cm	0.82ft
Axle Radius (r)	0.3cm	0.0098ft (0.118in)
Axle Span (L)	10cm	0.328ft (3.94in)
Axle Material 304 Steel Yield Strength	205 MPa	30,000psi

Sample Engineering Curriculum

Analyzing the Stress on the Trebuchet Axle continued

Follow these steps to use the flexural formula to answer the question “Will it break?”

1. Run the GEARS-TrebStar simulation using the given parameters.
2. Record the value for (P), the maximum (*Dynamic*) load on the axle from the GEARS-TrebStar trebuchet simulator. The maximum dynamic load value, **max(Fax/Fcw) =**, can be found on the lower right corner of the GEARS-TrebStar interface. *See the red circle in the adjacent figure.* The **max(Fax/Fcw)** value is the ratio of the maximum dynamic load to the counterweight mass. To calculate the maximum dynamic load, multiply the **max(Fax/Fcw)** value by the counterweight.
3. Use the flexural formula to calculate the maximum allowed stress, S_{max} for the specified axle radius (r). *Note: L and r are given in the preceding parameters list.*
4. Compare the calculated maximum allowed stress for the axle to the given yield strength for the type 304 steel axle material.



Example: An Explanation of the Preceding Problem.

1. Run the trebuchet design parameters through the GEARS-TrebStar simulator to determine the maximum dynamic load ratio, **max(Fax/Fcw) = 3.5**. When this value is multiplied by the counterweight the resulting maximum dynamic force on the axle is $3.5 \times 2.2\text{lbs} = 7.7\text{lbs}$ (P).
2. The axle radius (r) is 0.118 inches.
3. The Axle span is (L) is 3.94 inches, rounded off to 4 inches.
4. Substitute the values for P, r and L in the flexural equation to determine the maximum allowable stress for this axle system.

$$S_{max} = \left(\frac{1}{\pi}\right) * P * \frac{L}{r^3}$$

$$S_{max} = \left(\frac{1}{3.142}\right) * 7.7\text{lbs} * \frac{4\text{in.}}{0.118\text{in}^3}$$

$$S_{max} = 6126.7\text{lbs/in}^2$$

5. Comparing the allowable stress for the axle (6126.7psi) to the yield strength of the material (30,000psi) indicates an axle design with a margin of safety of nearly 5:1. This axle is certainly adequate for this application.

Sample Engineering Curriculum

Analyzing the Performance of the Same Axle Using SolidWorks COSMOSXpress

SolidWorks offers a handy utility that can be used to quickly determine if and how a trebuchet axle, or other load-bearing component will perform when stressed. COSMOSXpress can help answer questions like:

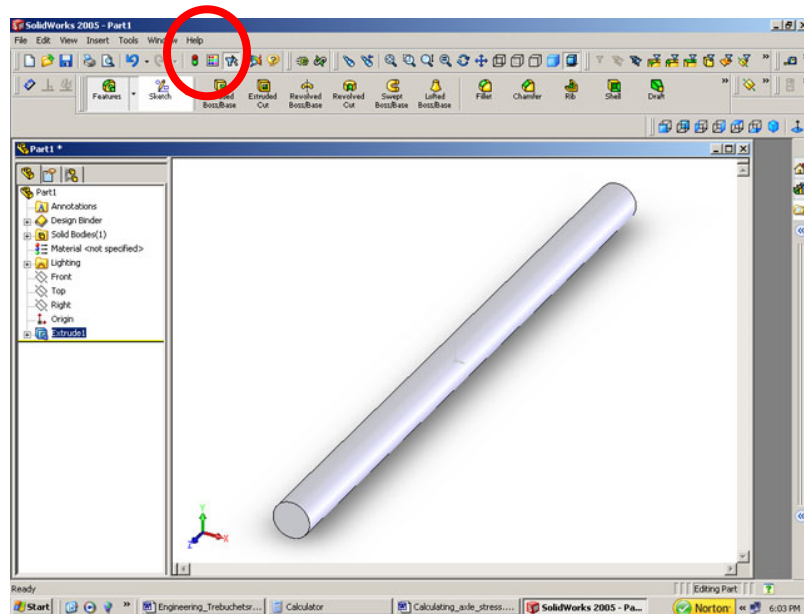
- Will the axle fail or break?
- How will it deform or bend?
- Can the axle dimensions be reduced and still support the applied loads?

COSMOSXpress is also a great tool that can be used to verify the answers you calculated using the flexural formula.

The icon for the COSMOSXpress utility can be found on the upper level of the SolidWorks toolbar.

Using COSMOSXpress is easy and self explanatory:

1. Sketch and extrude the axle on the front plane with the dimensions: $r = 0.118"$, $L = 4"$.
2. Click on the COSMOSXpress Icon.
3. Choose **Options/English**
4. Choose **Next/Steel/304** and **Apply** the material.
5. Choose **Next**, read the statement and choose **Next** again.
6. Select the round face on each end. This is a restraint set. You have essentially supported the axle on each end. Click **Next**.
7. Read the statement about force and pressure, select **Next**, and select the **Force** radio button.
8. Select the outside of the axle to add the **Load Set**. Click **Next**.
9. Click **Normal to a Reference Plane** radio button. Click on the **Top Plane** Specify 7.7 lbs. for the force value. Click **Next** and Click **Next** again.
10. Click the **Yes** radio button to accept the default settings.
11. Click **Next** and click **Run**. COSMOSXpress will calculate the lowest factor of safety. A factor of safety greater than 1 indicates the part will handle the design loads. Note: COSMOSXpress will usually yield a higher factor of safety than you will obtain using the flexural formula. This is due to the fact that the flexural formula assumes a worse case, center loading of the axle. This concentrates the force on the weakest part of the axle (Center span). COSMOSXpress on the other hand assumes a load that is equally distributed across the span. In actual practice, the real loading on a trebuchet axle is likely somewhere between the two values.



Sample Engineering Curriculum

- Click **Show Me**, after the calculations are completed. COSMOSXpress will show a color coded model of the axle. Blue indicates the axle is not being stressed near the material yield point, and that the factor of safety is greater than 1. Red indicates a factor of safety less than 1, or more importantly, that the part will fail under the applied load.
- After clicking **Show Me**, click **Next**, and choose “Show me the stress distribution in the model” **radio button**. This feature provides a color coded graphic showing where the part is most highly stressed. Warmer colors indicate higher stress and cooler colors indicate lower stress.

The axle in this example maintained a factor of safety greater than 2 under a load of 60 lbs. This is more than 7 times the maximum loading predicted by the GEARS-TrebStar simulation software.

COSMOSXpress Challenge

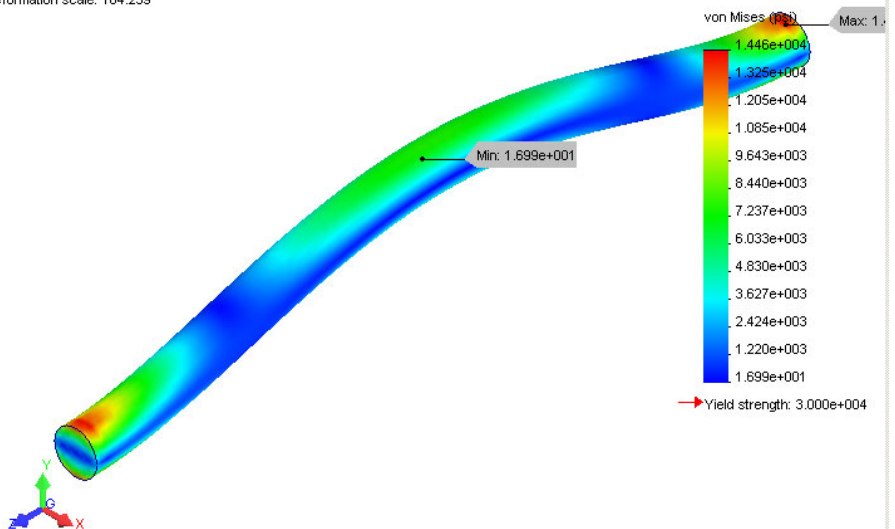
Using COSMOSXpress determine the minimum axle radius needed to support the maximum load imposed by the trebuchet used in these examples.

Hint: The maximum stress for this example is 7.7 pounds.

Answer

_____ inches

Model name: Part1
Study name: COSMOSXpressStudy
Plot type: Static nodal stress Plot1
Deformation scale: 164.239



Stress Distribution of 0.118" radius axle under 60 pound load. Courtesy of SolidWorks Corp.

Try This

Use your new engineering skills and knowledge to help design a real trebuchet. This may take some time, and more than a few iterations to find the best answer, but you have the tools and skills to create the design.

Tools you will need to use:

Calculator
Flexural Formula
COSMOSXpress analysis software
GEARS-TrebStar simulation software

Trebuchet Design Parameters

Maximum Dynamic Load (P) = 20 lbs (9kg)
Span (L) 6 in. (15cm)

Use the constraints imposed by the trebuchet design parameters listed above to design a trebuchet axle with the minimum radius necessary to support the Maximum Dynamic Load (P) with a factor of safety of 2.5.

Record all your work in your notebook including the values and number of design iterations you created. Sign and date your work.

Engineering Trebuchets

Visualizing and Describing The Engineering of Totally Trebuchet

This rubric is offered as an example. Teachers and students are encouraged to create assessment tools that reflect their needs and expectations.

Required Skill or Competency	Meets or Exceeds this Requirement Score 4-5 pts	Meets Some of this Requirement Score 2-3 pts	Meets little or None of this Requirement Score 0-1 pts
Explain how forces and energy in a trebuchet system are affected by scaling.			
Calculate the maximum theoretical range of a trebuchet given the drop height of the counterweight and the masses of the counterweight and projectile.			
Calculate the range efficiency of the trebuchet.			
Use the TrebStar computer simulation to design and record the performance of a full sized trebuchet with a uniform beam.			
Explain and describe the engineering process in writing.			
Use the engineering process to invent a device or process to improve the performance of a model trebuchet.			
Use a calculator or SolidWorks to find the moment of inertia about the axle of a uniform beam of given dimensions and mass.			
Calculate the radius of gyration of a trebuchet beam given the distance between the point mass and axle.			
Test and graph the relationship between any of the eight trebuchet independent variables (Parameters) and a dependent variable.			
Design a tapered trebuchet beam. Use SolidWorks to create the models and calculate the volume, mass and center of gravity.			
Use the CM and RG values calculated by Don Siano to make simulated hurls based on the dimensions and masses of the GEARS model trebuchet.			
Build Report/Notebook assessment			